UC SANTA BARBARA

Information and Influence:
Overcoming and Exploiting Uncertainty in Congestion Games

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For the ControlX series at the University of Washington

UNIVERSITY of WASHINGTON

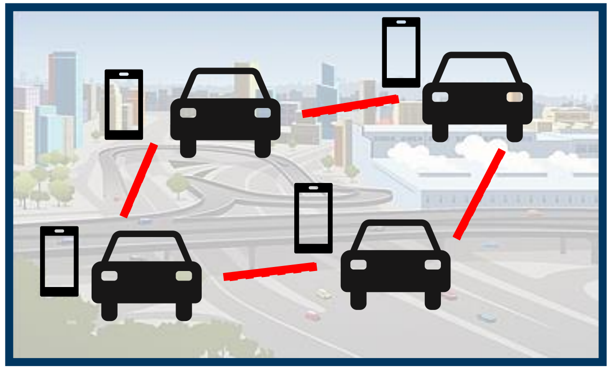
Control in Large-Scale Systems

New Communication Technology

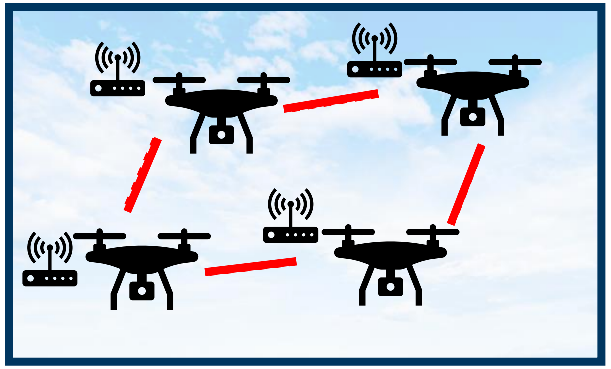
↓

New Control Opportunities

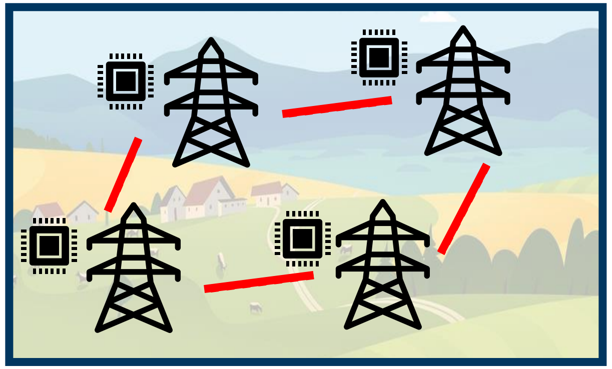
Traffic Network



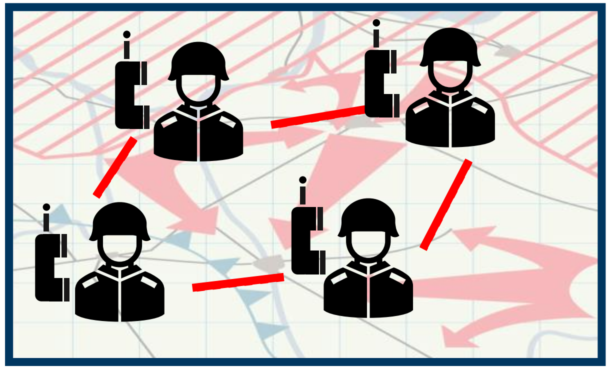
Robot Fleet



Power Grid



Defense Allocation



Emergent problems:

- Cannot ***directly*** control ***every*** component
- ***Information*** affects control capabilities

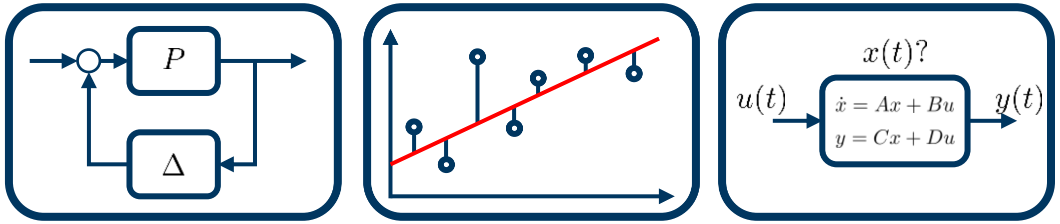
Information and Uncertainty in Control

Uninformed designer must ***overcome*** uncertainty ***in*** control

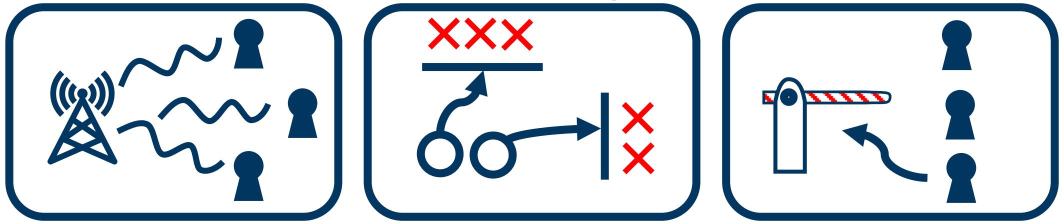
Flip Information Paradigm

Informed designer can ***exploit*** uncertainty ***as*** control

Robust Control | Estimation | Observability

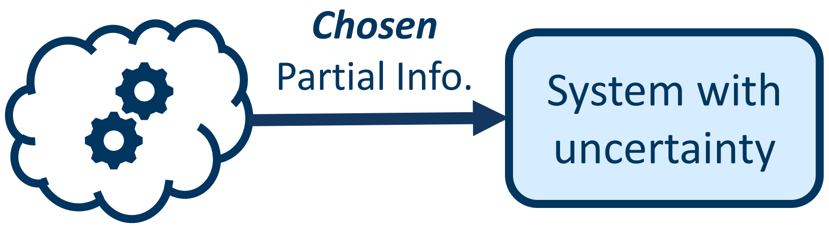


Signalling | Maneuver/ Posturing | Admission



Uncertain Parameters

Control Design



Control of Information

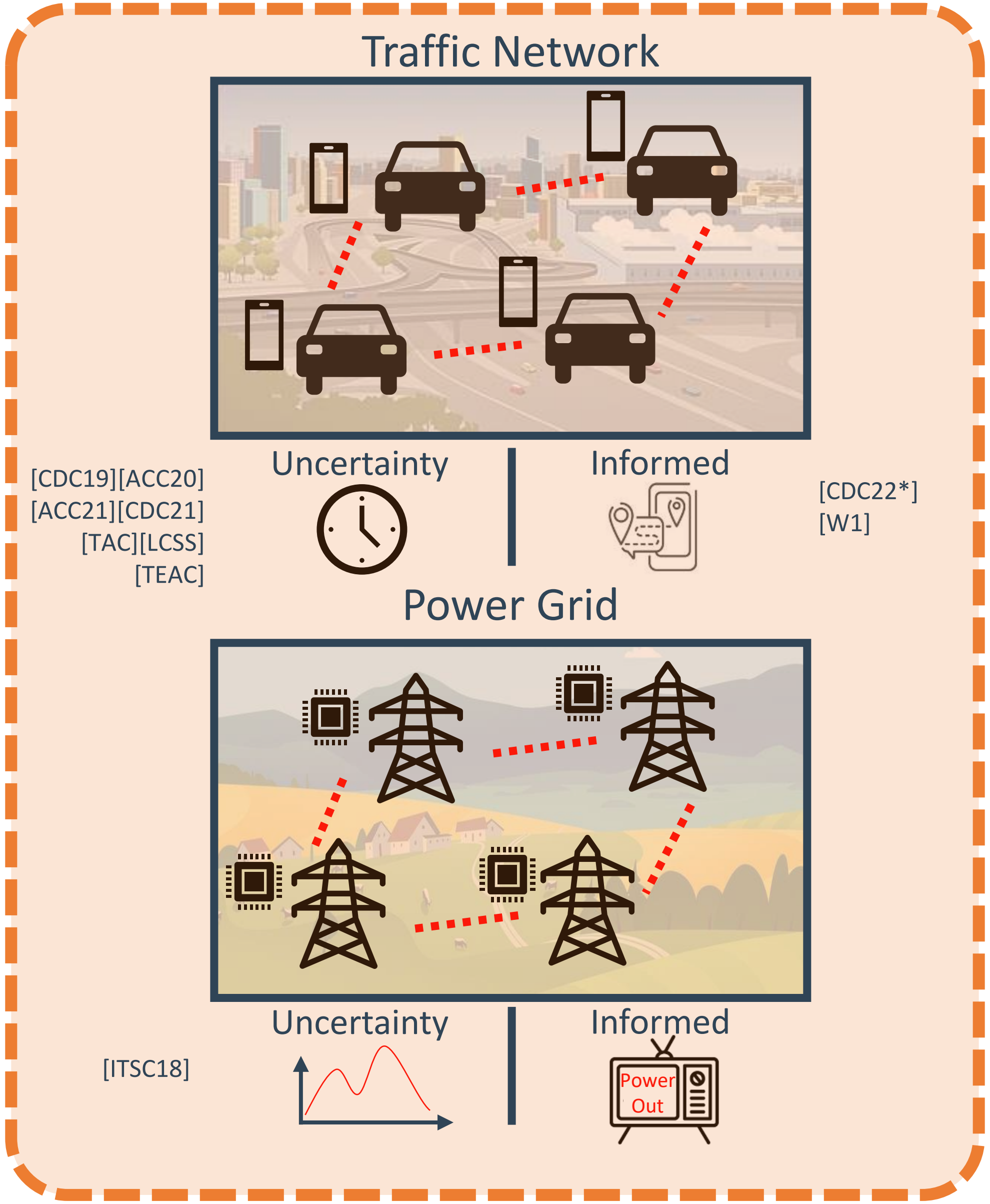
Need to design ***robust*** to ***uncertainty***

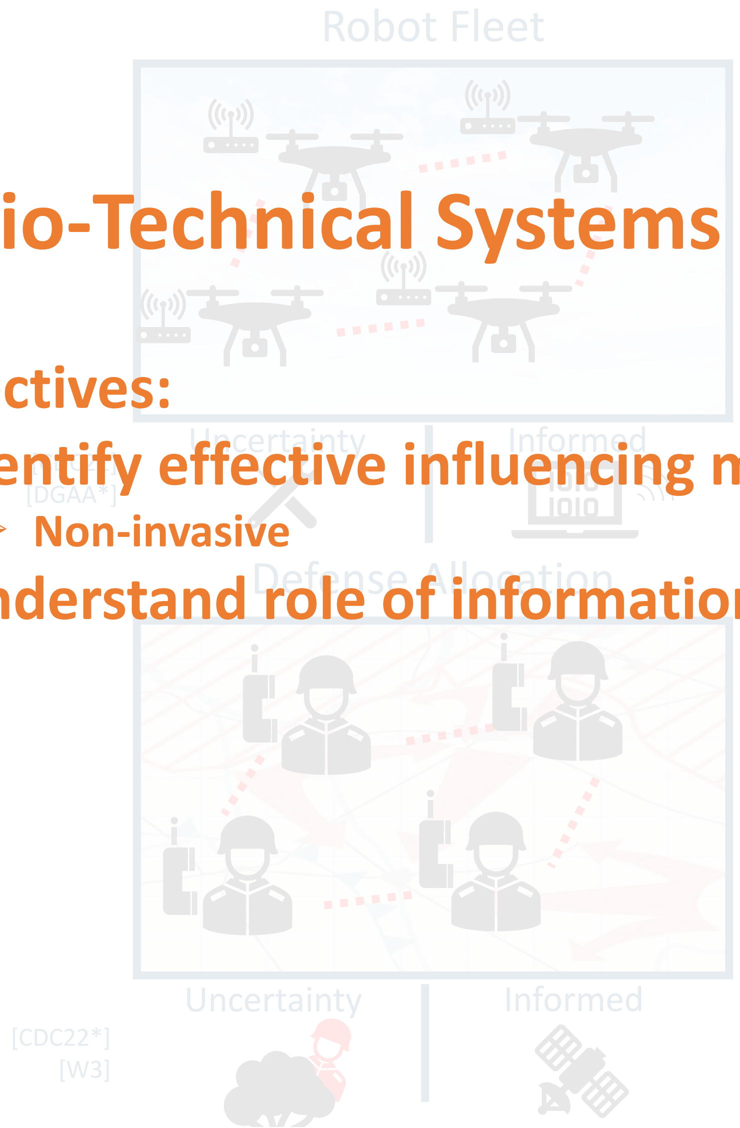
Not sharing info can be ***bad***

Sharing ***all*** info can be ***bad***

Need to share info ***intelligently***

Control in Large-Scale Systems



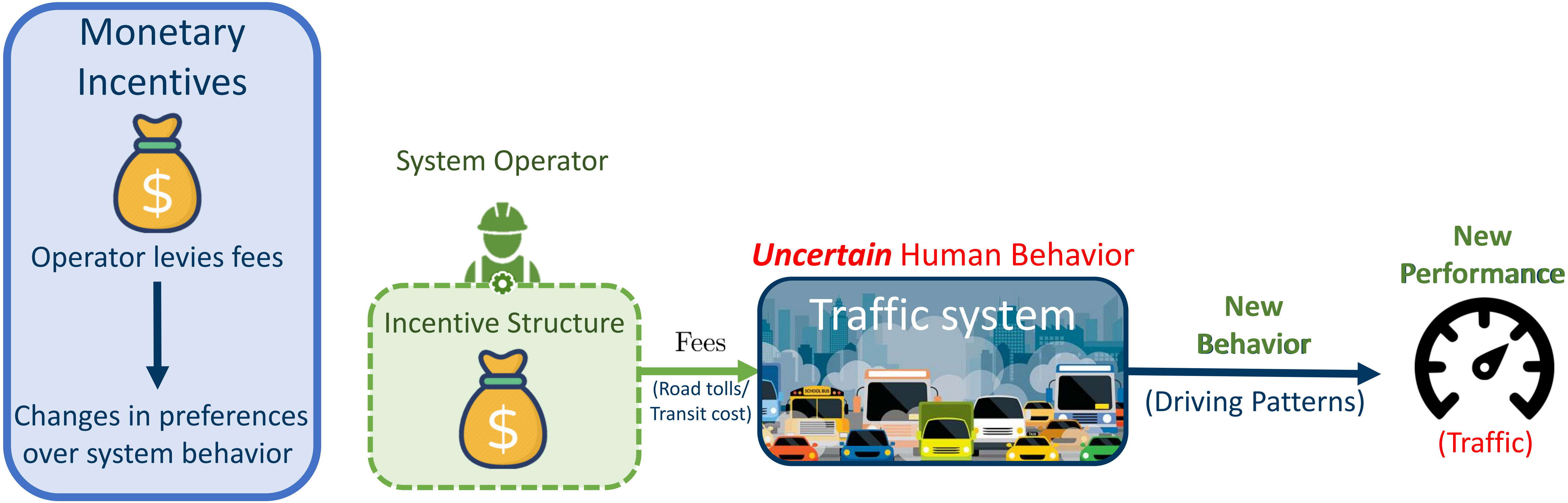


Socio-Technical Systems

Objectives:

- Identify effective influencing mechanisms
  - ➢ Non-invasive
- Understand role of information/uncertainty

1. Monetary Incentives



Q?:

1. How to ***design*** with ***uncertainty***?
2. How ***information*** affects ***performance?***

A:

1. ***Robust*** incentives
2. Performance guarantees

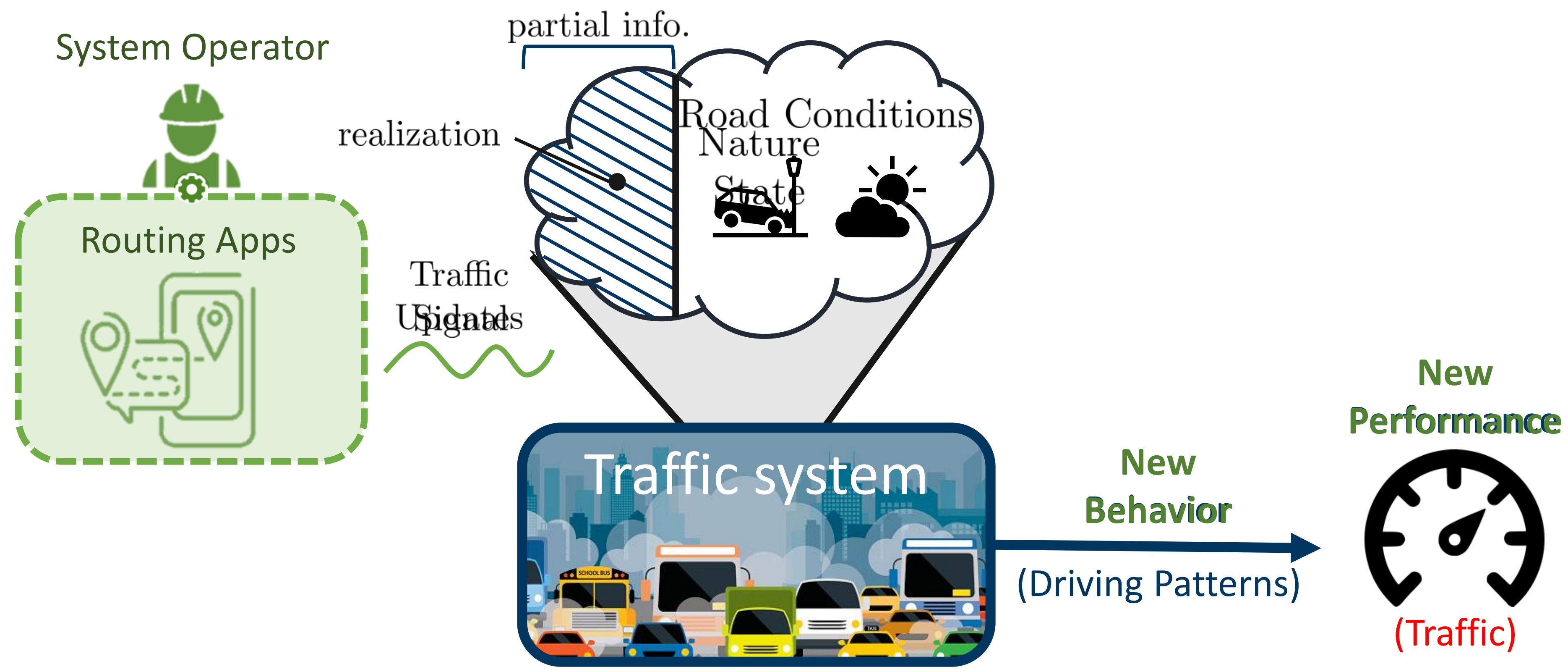
2. Information Signalling

Information Signalling

Operator observes realization of state

↓

Partially signal information to agents



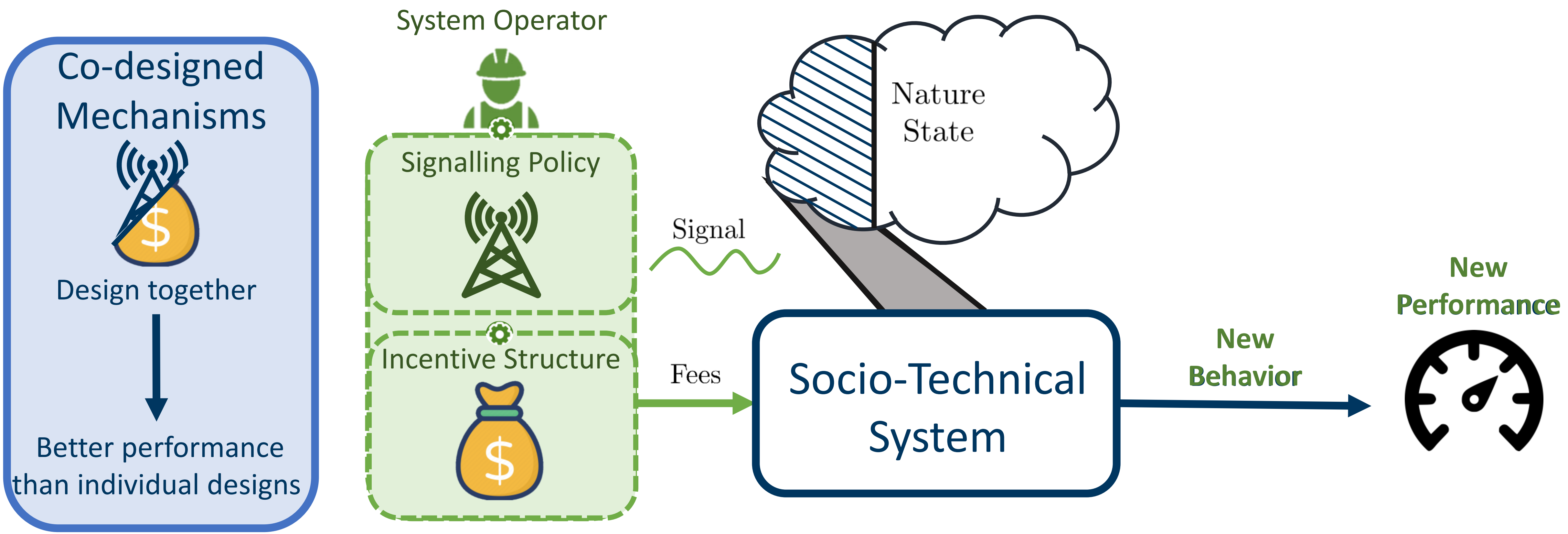
Q?:

1. Is signalling effective?
2. How to design ***signalling policy***?

A:

1. Has the potential to ***help*** or ***hurt***
2. Methods to solve for optimal signal

3. Incentive-Signal Co-design



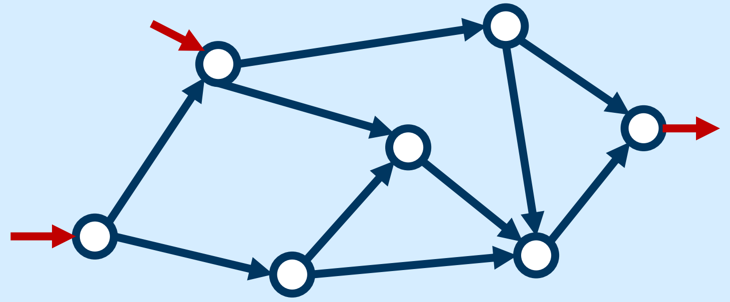
Q?:
1. ***Benefit*** to designing ***concurrently***?
2. How to ***co-design*** mechanisms?

A:
1. Incentives ***robustify*** signalling
2. Methods to solve co-design

Relevant Problem Features

1. Users with ***individual decision making***
2. Actions aligned with ***relevant system behavior***
3. Users' ***decisions affect*** the system and ***each other***

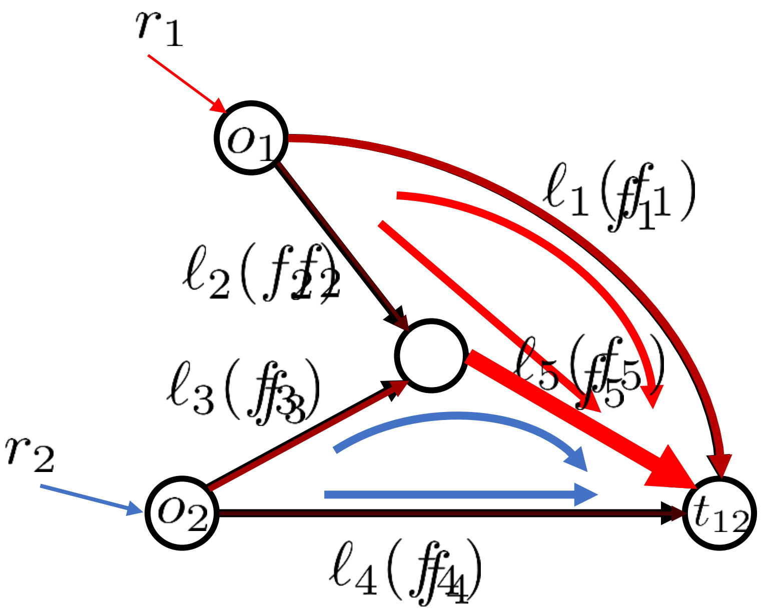
Network Congestion Games



Network Congestion Games

Routing Problem G

- Graph (V, E)
- Origin-destination pairs (o_i, t_i)
  - Mass of traffic r_i
- User $x \in [0, r_i]$ selects a path $P_x \in \mathcal{P}_i$
- Flow $f = \{f_e\}_{e \in E}$
- Latency functions $\ell_e(f_e)$
  - Non-decreasing, cont. diff.



- Cost minimizing users

$$P_x \in \arg\min_{P \in \mathcal{P}_i} J_x(P; f^{\mathrm{Nf}}) = \sum_{e \in P} \ell_e(f_e^{\mathrm{Nf}}) \ \forall x \in N$$

Nash/user/Wardrop flow f^{Nf}

Emerge from many natural learning dynamics and essentially unique

Are these good states to be at?

System Performance

System Cost = Social Welfare

Total Latency

$$\mathcal{L}(f) = \sum_{e \in E} f_e \ell_e(f_e)$$

aggregate/average user travel time

Optimal Flow

$$f^{\text{opt}} \in \underset{f \text{ is feasible}}{\arg\min}\ \mathcal{L}(f)$$

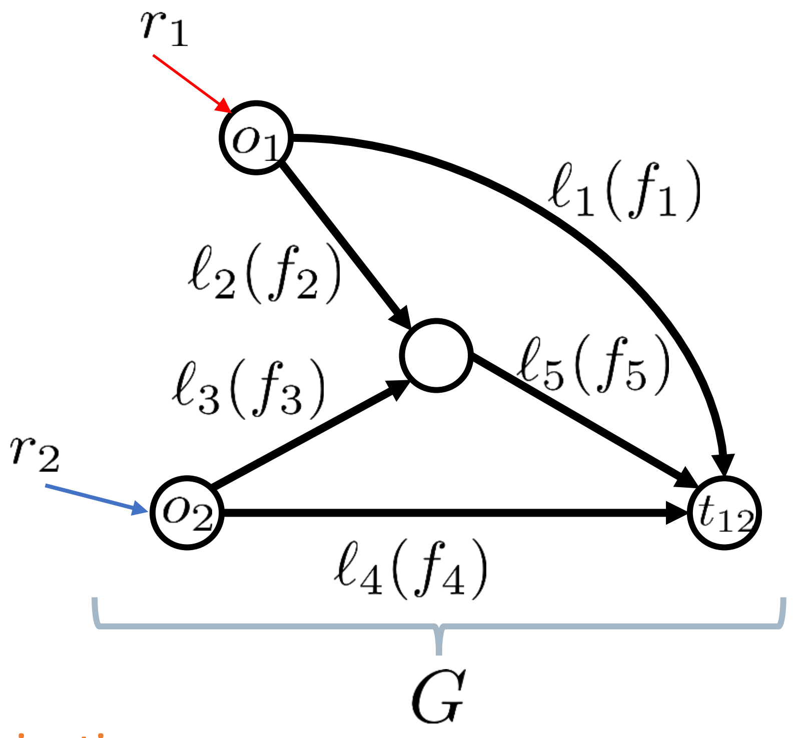
Compare selfish to optimal

Price of Anarchy

$$\mathcal{L}(f^{\text{Nf}}) \quad \text{VS} \quad \mathcal{L}(f^{\text{opt}})$$

$$\text{PoA}(G) := \frac{\mathcal{L}(f^{\text{Nf}})}{\mathcal{L}(f^{\text{opt}})} \geq 1$$

Measure for inefficiency of selfish routing

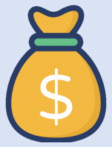


Objective:

Understand influencing mechanisms' abilities to reduce inefficiency

Incentive Mechanisms

Monetary Incentives



Operator levies fees on user actions → Changes in preferences over system behavior

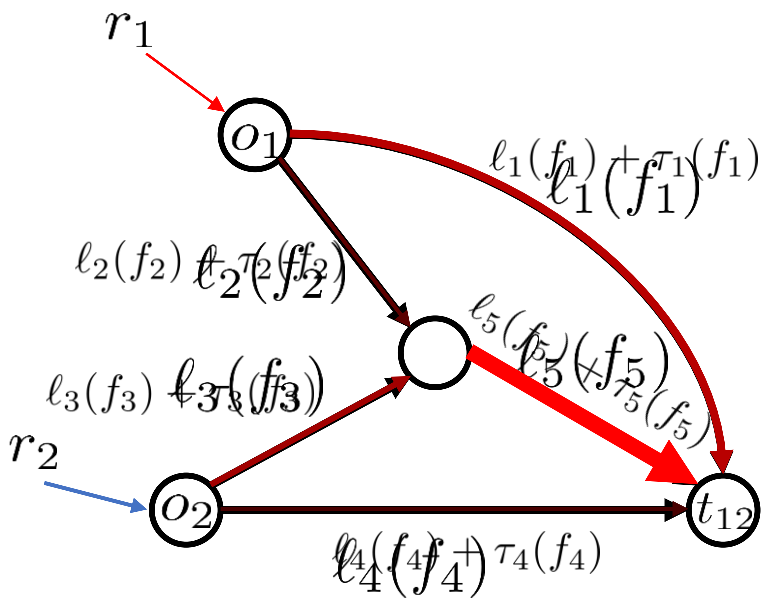
Incentive mechanism

$$T(G) = \{\tau_e : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}\}_{e \in E}$$

User cost

$$\underbrace{J_x(P_x; f, T) = \sum_{e \in P_x} \ell_e(f_e) + \tau_e(f_e)}$$

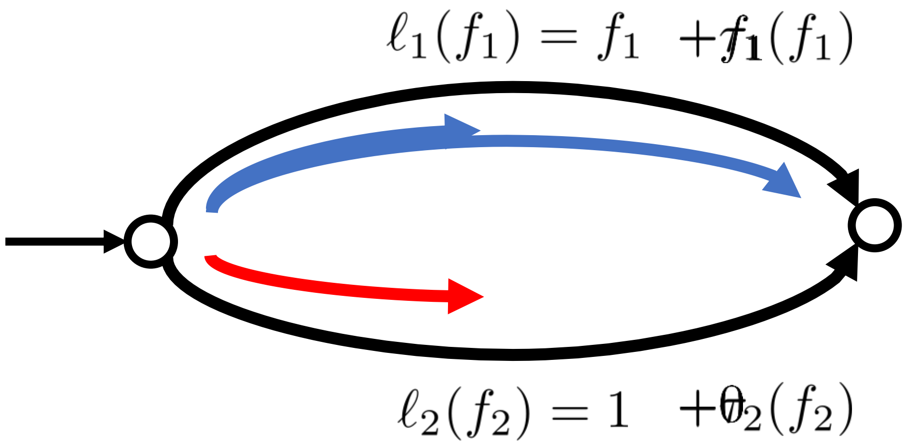
New Nash flow f^{Nf} → New system cost



Objective:

$$T^{\mathrm{opt}} \in \arg\min_{T} \ \mathrm{PoA}(G, T)$$

Example



Total Latency

$$\mathcal{L}(f) = \sum_{\text{edges}} f_e \ell_e(f_e)$$

Optimal Flow

$$\mathcal{L}(f^{\mathrm{opt}}) = \frac{3}{4}$$

Selfish Routing: Nash Flow

$$\mathcal{L}(f^{\mathrm{Nf}}) = 1 \quad \frac{3}{4}$$

Price of Anarchy:

$$\frac{\mathcal{L}(f^{\mathrm{Nf}})}{\mathcal{L}(f^{\mathrm{opt}})} = 1 \quad \frac{4}{3}$$

How does ***uncertainty*** affect our ability to incentivize?

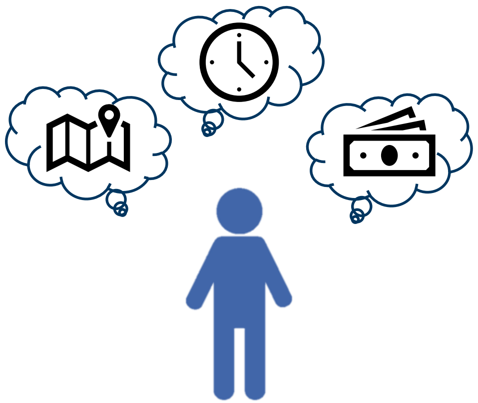
Uncertain User Response

We can ***not*** perfectly predict how users respond to incentives

Each user has unknown price-sensitivity $s_x \in [S_L, S_U]$

$$J_x(P_x, f) = \sum_{e \in P_x} \ell_e(f_e) + \underbrace{s_x}_{\text{Value of time vs money}} \tau_e(f_e)$$

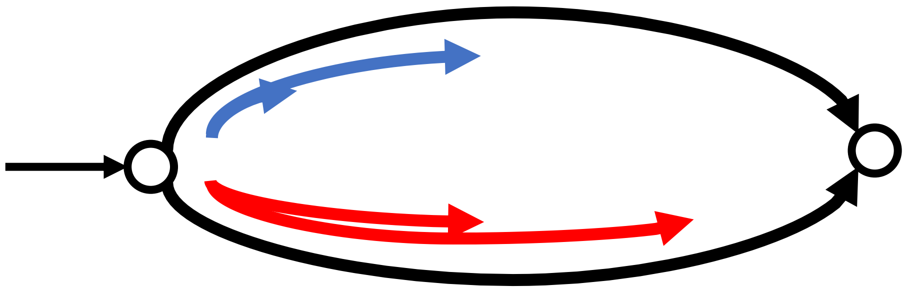
Value of time vs money



Population sensitivity distribution

$$s \in \mathcal{S} = \{s : N \to [S_\mathrm{L}, S_\mathrm{U}]\}$$

Example



Highly sensitive users: $s_x = 10 \; \forall x \in N$

How do we design incentives with ***uncertainty*** about ***price sensitivities***?

Total Latency

$$\mathcal{L}(f) = \sum_{\text{edges}} f_e \ell_e(f_e)$$

Optimal Flow

$$\mathcal{L}(f^{\text{opt}}) = \frac{3}{4}$$

Selfish Routing: Nash Flow

$$\mathcal{L}(f^{\text{Nf}}) = 0.91$$

Price of Anarchy:

$$\frac{\mathcal{L}(f^{\text{Nf}})}{\mathcal{L}(f^{\text{opt}})} \approx 1.213$$

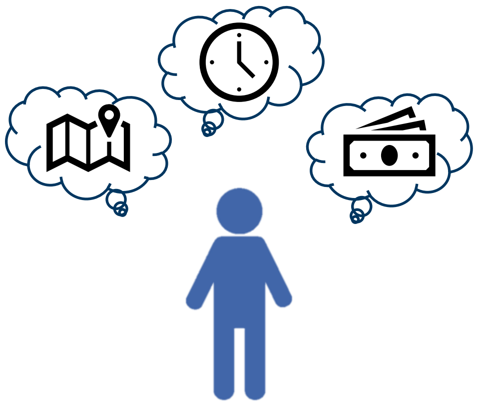
Uncertain User Response

We can ***not*** perfectly predict how users respond to incentives

Each user has unknown price-sensitivity $s_x \in [S_L, S_U]$

$$J_x(P_x, f) = \sum_{e \in P_x} \ell_e(f_e) + s_x \tau_e(f_e)$$

Value of time vs money



Population sensitivity distribution

$$s \in \mathcal{S} = \{s : N \to [S_\mathrm{L}, S_\mathrm{U}]\}$$

Worst case: No information (Knightian uncertainty)

$S_U / S_L \geq 1$ is a measure of uncertainty

$$T^{\mathrm{opt}} \in \arg\min_T \mathrm{PoA}(G, \mathcal{S}, T) = \sup_{s \in \mathcal{S}} \mathrm{PoA}(G, s, T)$$

Objective: Robust incentive design

Existing Results

Full info. [Fleischer, et. al.] [Cole, et. al.]

- Optimal incentives with heterogeneous price sensitive users

No info. [Brown, et. al.]

- Optimal tolls with heterogeneous price sensitive users and price of anarchy bound
    - Restricted incentives in limited setting

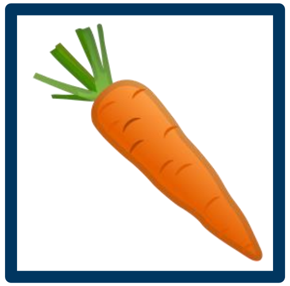
Today:

- Value of information
- Budget constraints
- Different incentive types

Subsidies and Tolls

Though we could use both…
Consider separately to determine important qualities of each

Tolls



Subsidies

Tolling function:

$$\tau_e^+(f_e) \geq 0 \quad \forall f_e \geq 0$$

Tolling mechanism:

$$T^+(e;G) = \tau_e^+$$

Only assigns tolls

Optimal tolling mechanism:

$$T^{\text{opt}+} \in \arg\min_{T^+} \text{PoA}(G, T^+)$$

Subsidy function:

$$\tau_e^-(f_e) \leq 0 \quad \forall f_e \geq 0$$

Subsidy mechanism:

$$T^-(e;G) = \tau_e^-$$

Only assigns subsidies

Optimal subsidy mechanism:

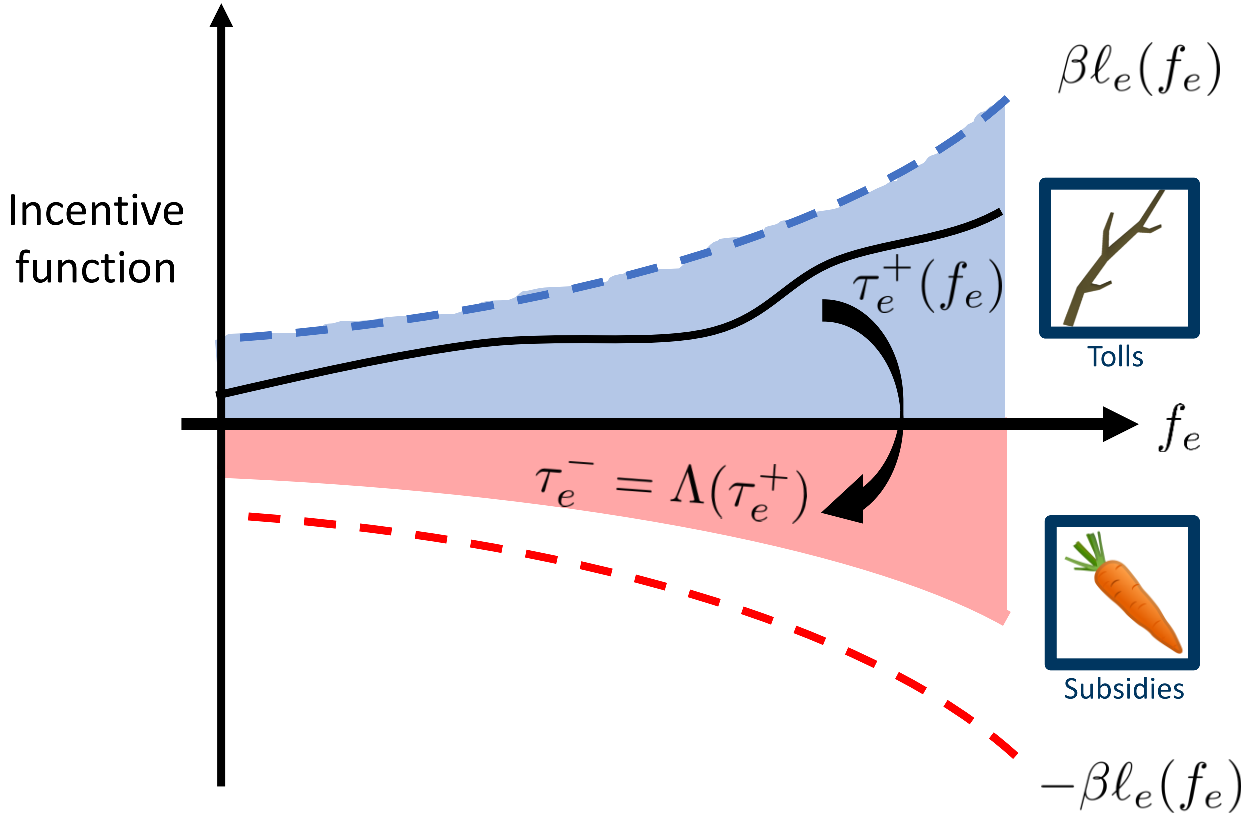
$$T^{\text{opt}-} \in \arg\min_{T^-} \text{PoA}(G, T^-)$$

Tolls
$\text{PoA}(G, T^{\text{opt}+})$

Subsidies
$\text{PoA}(G, T^{\text{opt}-})$

Budgetary Constraints

Added Constraint: $|\tau_e(f_e)| \leq \beta \ell_e(f_e) \quad \forall f_e \geq 0$



Full info/homogeneous (i.e., $S_L = S_U = 1$)

Theorem 1.1 [ACC20,LCSS,TAC]

For a family of congestion games $\mathcal{G}$, under bounding factor $\beta \geq 0$,

$$\mathrm{PoA}(G, T^{\mathrm{opt}+}(\beta)) \geq \mathrm{PoA}(G, T^{\mathrm{opt}-}(\beta)) \geq 1.$$

Additionally, if the budget constraint is active for every optimal incentive, the inequalities are strict.

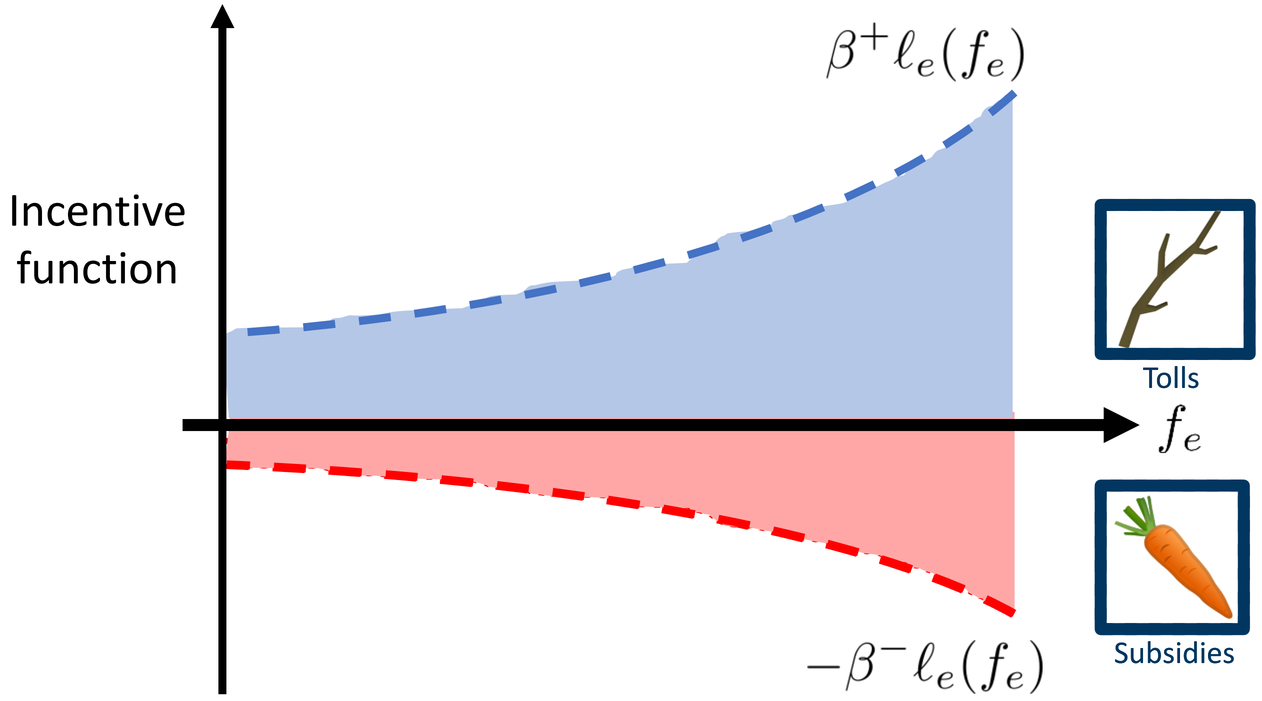
Smaller subsidies can outperform ***larger tolls***.

Budgetary Constraints & User Heterogeneity

What happens when we introduce uncertainty into the problem? No info/heterogeneous (i.e., $s_x \in [S_\mathrm{L}, S_\mathrm{U}]$)

Start with *nominally equivalent* bounded subsidies and tolls, i.e.,

$$\mathrm{PoA}(\mathcal{G}, T^{\mathrm{opt}+}(\beta^+)) = \mathrm{PoA}(\mathcal{G}, T^{\mathrm{opt}-}(\beta^-))$$ when users are homogeneous.



As user become heterogeneous:

Theorem 1.2 [ACC20,LCSS,TAC]

For a congestion game G, under bounding factors β^+, β^- respectively, with possible price-sensitivity distributions $\mathcal{S}$,

$$\mathrm{PoA}(G, \mathcal{S}, T^{\mathrm{opt}-}(\beta^-, \mathcal{S})) \geq \mathrm{PoA}(G, \mathcal{S}, T^{\mathrm{opt}+}(\beta^+, \mathcal{S})) \geq 1.$$

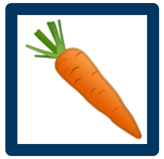
Additionally, if G is responsive to user heterogeneity, the inequalities are strict.

Performance of ***subsidies is less robust*** to player heterogeneity than tolls.

Effect of Uncertainty

Significant budgetary constraint

Thm. 1.1



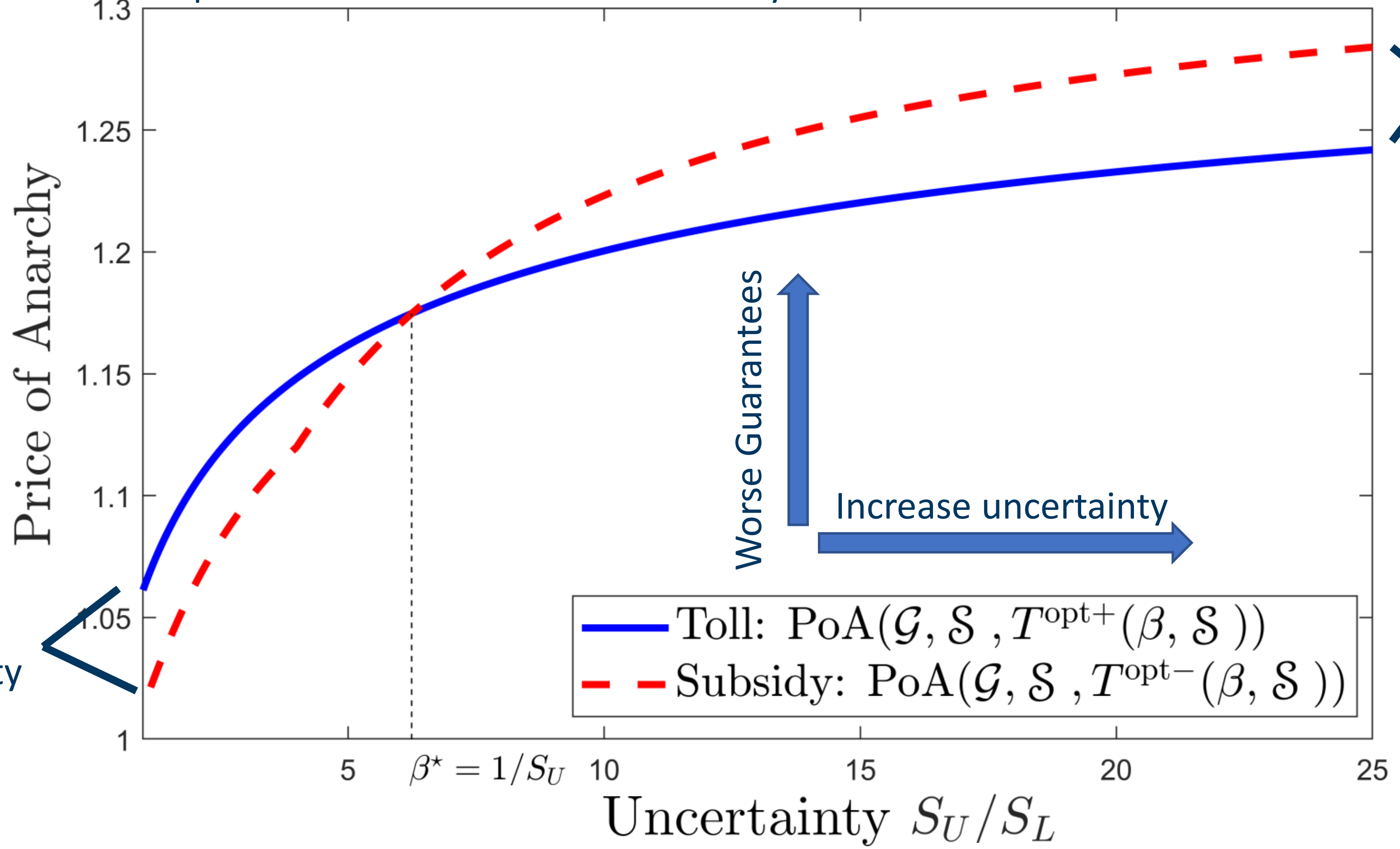
Subsidies

Significant user heterogeneity

Tolls

Thm. 1.2

In parallel networks with affine latency functions



Tolls are more robust than subsidies

Subsidies outperform tolls when high certainty

Other Contributions

- Further uncertainty over network structure/latency functions

- Partial information
  - How do pieces of information help improve performance? [CDC19,TCSS*]

- Fairness vs performance
  - How does improving performance affect fairness? [ACC21]

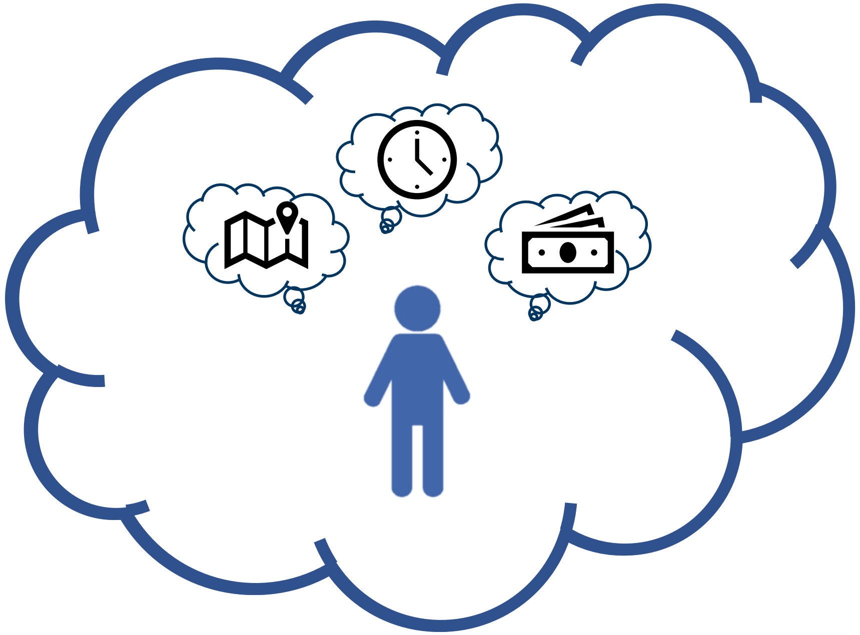
- Unincentivizable users
  - What if some users do not receive incentives? [CDC21]

Users' Uncertainty

Uncertainty for system operator

Uncertainty for system ***users***

?



Can users' ***uncertainty*** be ***exploited***?

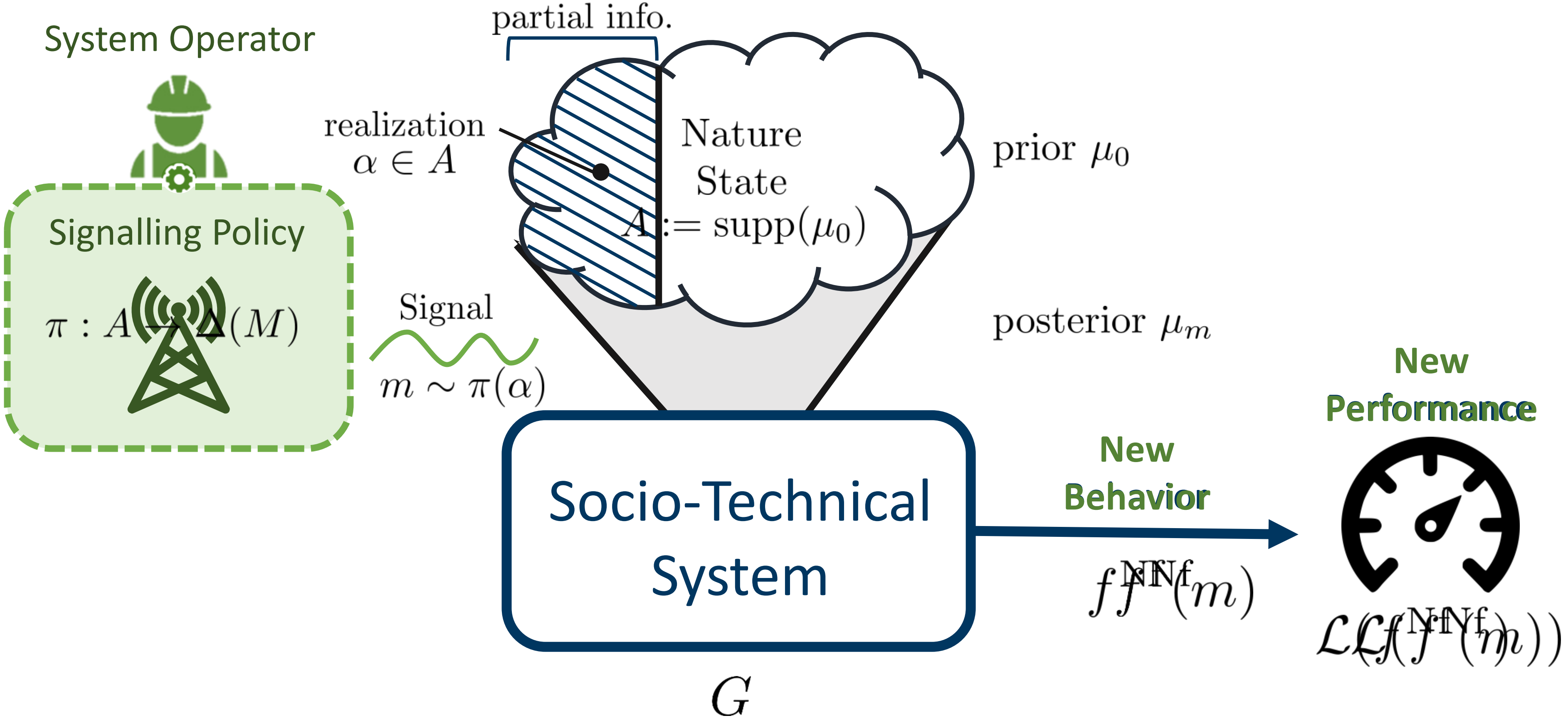
Information Signalling

Information Signalling

Operator observes realization of state

↓

Partially signal information to agents

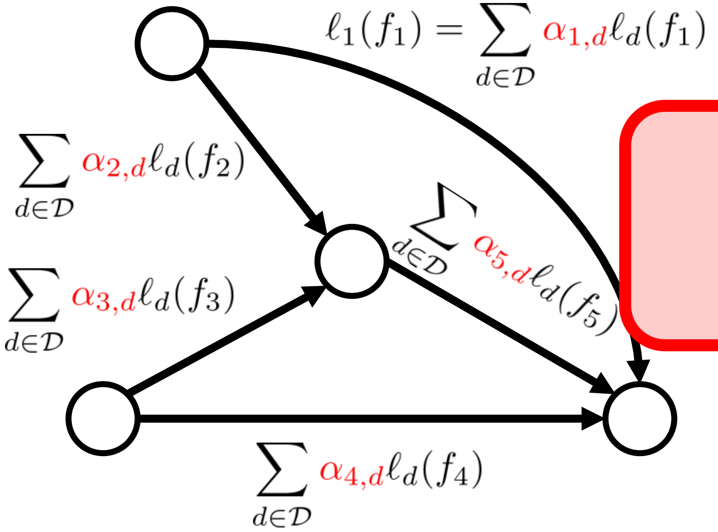


Revealing full info can ***hurt*** system performance

How do we signal intelligently?

Bayesian Congestion Game

Unknown State → Uncertain Congestion Rates: $\alpha = \begin{bmatrix} \vdots \\ \alpha_{e,d} \\ \vdots \end{bmatrix} \in \mathbb{R}_{\geq 0}^{|E| \cdot |\mathcal{D}|}$



Latency function:

$$\ell_e(f_e) = \sum_{d \in \mathcal{D}} \alpha_{e,d} \ell_d(f_e)$$

Basis Latency functions

e.g., $\mathcal{D} = \{1, f_e^2, e^{f_e}\}$

Prior belief:

$x]$

)

n) → (distribution of signal)

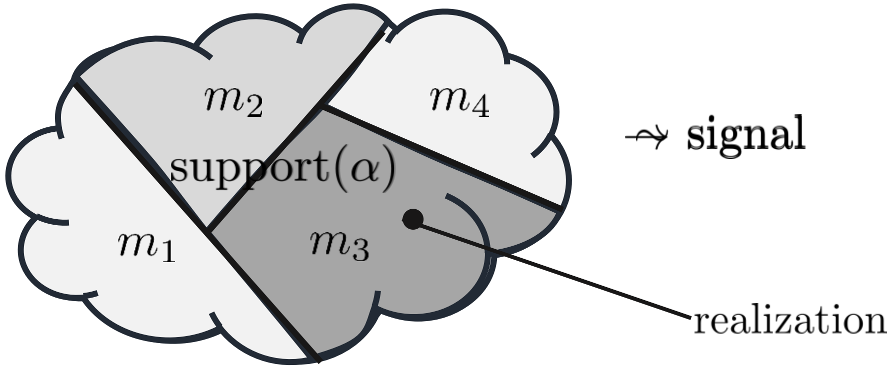
What does signalling do to system behavior?

Posterior belief

$\mu_m(x) = \mathbb{P}[\alpha = x | m]$

E.g., truthful

General



Efficacy of Signalling

Information Signalling

Operator observes realization of state → Partially signal information to agents

Given signal $\pi : A \to \Delta(M)$ and prior μ_0

Bayesian-Nash flow $\mathbf{f}^{\text{BNf}} = \{f(m)\}_{m \in M}$

agents pick an edge based on received signal

Expected User Cost

$$J_x(P_x; f(m)) = \mathbb{E}\left[\sum_{e \in P_x} \ell_e(f_e(m)) \mid m\right]$$

System Cost: *Expected* Total Latency in a BNf

$$\mathcal{L}^{\text{BNf}}(\pi; \mu_0) = \mathop{\mathbb{E}}_{\alpha}\left[\mathcal{L}(\alpha, f(m))\right]$$

Performance metric: *Benefit of Signalling*

$$\mathbf{B}(\pi; \mu_0) = \underbrace{\mathcal{L}^{\text{BNf}}(\emptyset; \mu_0)}_{\text{without signalling}} - \underbrace{\mathcal{L}^{\text{BNf}}(\pi; \mu_0)}_{\text{with signalling}}$$

Reduction in system cost from signalling

Benefit of Signalling

Can signalling ***help***?

Can signalling ***hurt***?

Proposition 2.1: There exists a signalling policy π in a Bayesian-congestion game G with prior μ_0 over the latency coefficient parameter α that has arbitrarily negative benefit, i.e.,

$$\inf_{\mu_0, \pi, G} \mathbf{B}(\pi; \mu_0) = -\infty.$$

Recall:

$$\mathbf{B}(\pi; \mu_0) < 0 \quad \Rightarrow \quad \mathcal{L}^{\mathrm{BNf}}(\pi; \mu_0) > \mathcal{L}^{\mathrm{BNf}}(\emptyset; \mu_0)$$

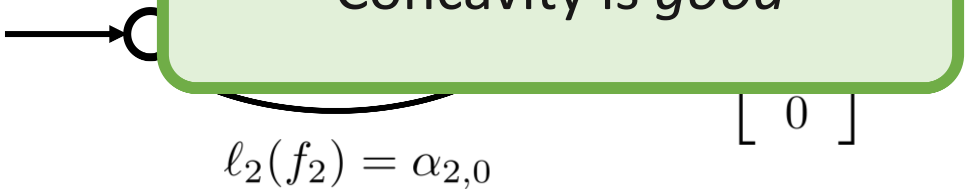
Note: Worst example comes from revealing full information

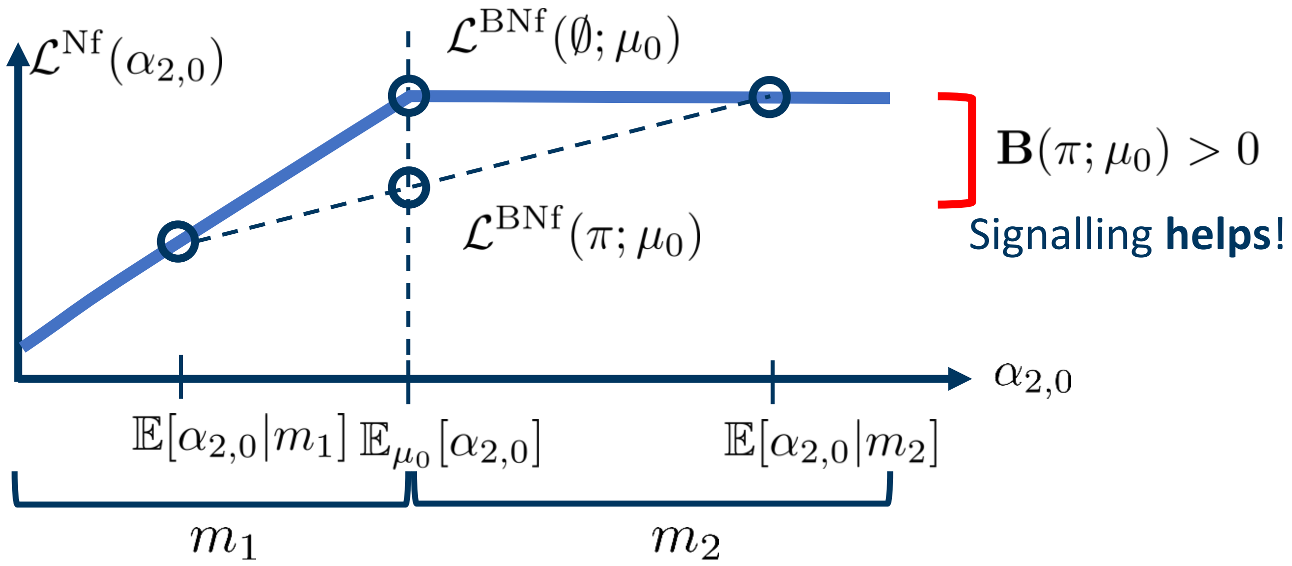
Illustrative example

Lemma: $\mathcal{L}^{\mathrm{BNf}}(\pi;\mu_0) = \sum_{m\in M} \psi(m)\mathcal{L}^{\mathrm{Nf}}(\mathbb{E}_{\alpha\sim\mu_m}[\alpha])$

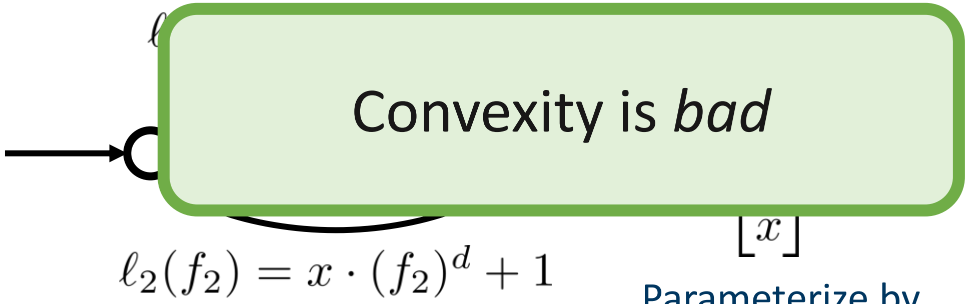
Total Lat. In Nash flow of deterministic C.G. with $\alpha = \mathbb{E}_{\alpha\sim\mu_m}[\alpha]$

Concavity is *good*

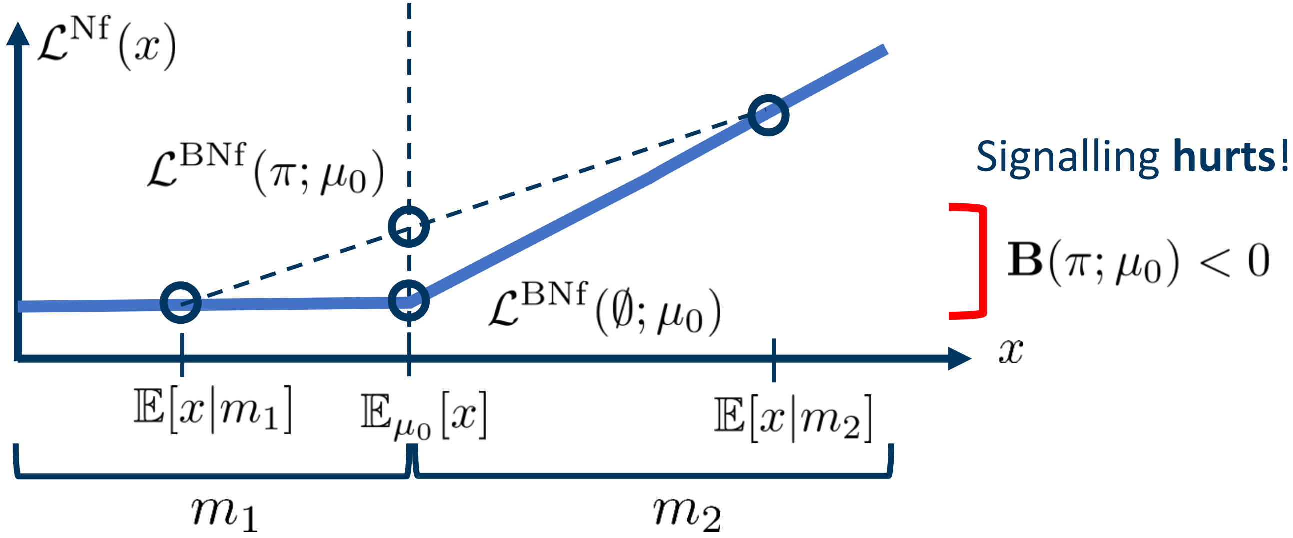




Convexity is *bad*



Parameterize by single variable x



Benefit of Signalling

How much can signalling ***help***?

Restrict to parallel networks and polynomial latency functions

i.e., $\ell_e(f_e) = \sum_{d \in \mathcal{D}} \alpha_{e,d}(f_e)^d \qquad \mathcal{D} = \{d_1, \ldots, d_{|\mathcal{D}|}\},\ d_i \in \mathbb{N}$

Theorem 2.2: For any set of polynomial degrees $\mathcal{D}$, prior μ_0, and signalling policy π:

$$-\underbrace{\sqrt{|\mathcal{D}|}}\cdot \underbrace{\|\mathbb{E}[\alpha] - \underline{\alpha}\|_2} \leq \mathbf{B}(\pi;\mu_0) \leq \underbrace{\sqrt{|\mathcal{D}|}}\cdot\underbrace{\|\mathbb{E}[\alpha] - \underline{\alpha}\|_2},$$

where $\mathbb{E}[\alpha] = \int_{x\in A} x \cdot \mu_0(x)dx$, and $\underline{\alpha} \in \mathbb{R}_{\geq 0}^{|E|\cdot|\mathcal{D}|}$ such that $\underline{\alpha}_{e,d} = \inf\{\text{supp}(\alpha_{e,d})\}$ for each $e \in E$, $d \in \mathcal{D}$.

Observations:

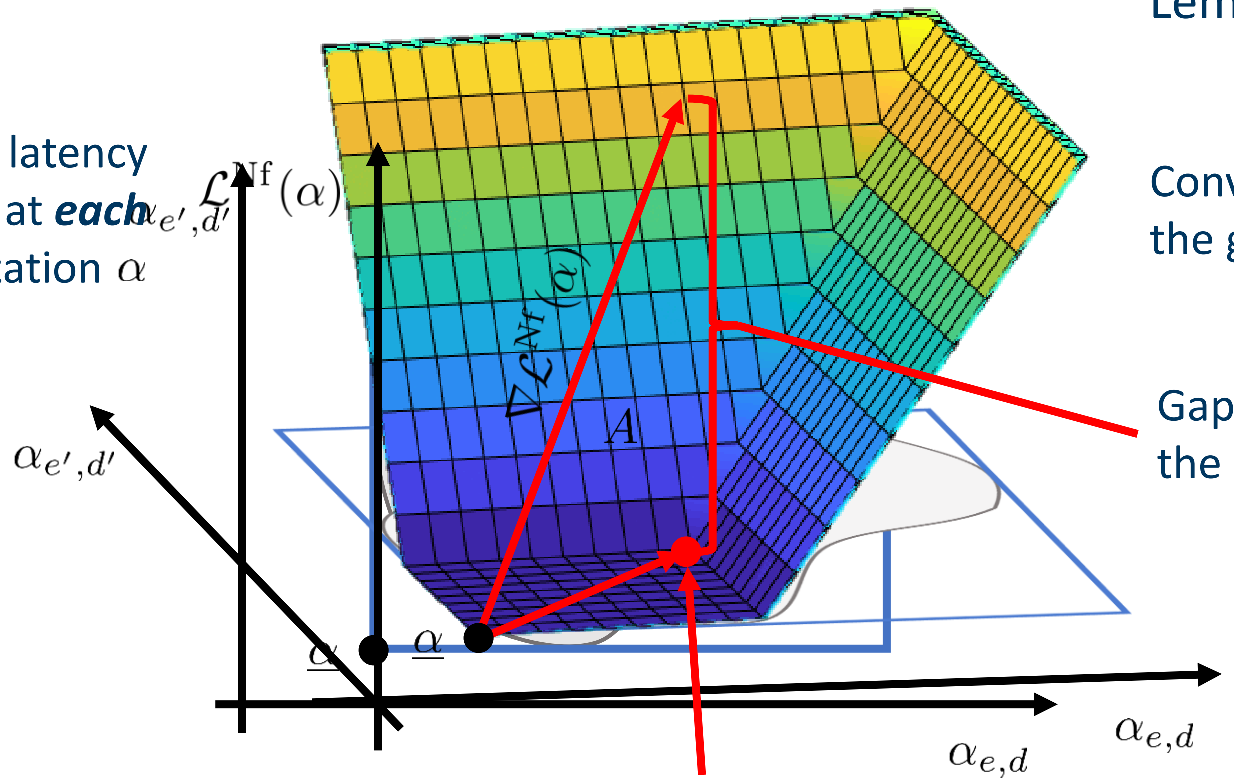
1. Signals can help *or* hurt performance
2. Bounds depend on
   I. Complexity of model
   II. Spread of α

Proof Sketch

Recall

Lemma: $\mathcal{L}^{\mathrm{BNf}}(\pi;\mu_0) = \sum_{m\in M} \psi(m)\mathcal{L}^{\mathrm{Nf}}(\mathbb{E}_{\alpha\sim\mu_m}[\alpha])$

Total latency at ***each*** Realization α



Convex hull of the graph of $\mathcal{L}^{\mathrm{Nf}}(\alpha)$

Gap between $\mathcal{L}^{\mathrm{Nf}}(\alpha)$ and the hull bound the benefit

Bound the gap with the gradient $\nabla\mathcal{L}^{\mathrm{Nf}}(\alpha)$

Without signalling
$\mathcal{L}^{\mathrm{BNf}}(\emptyset;\mu_0) = \mathcal{L}^{\mathrm{Nf}}(\mathbb{E}_{\mu_0}[\alpha])$

Insights on Signalling

- Signalling can have negative consequences
  - Negative benefit
- Identified good/bad situations to use signalling
  - Concavity/convexity
- Bound how effective signalling can be
  - In the context of parallel-network, polynomial-latency Bayesian congestion games

Can we do anything to ***ensure*** signalling ***helps***?

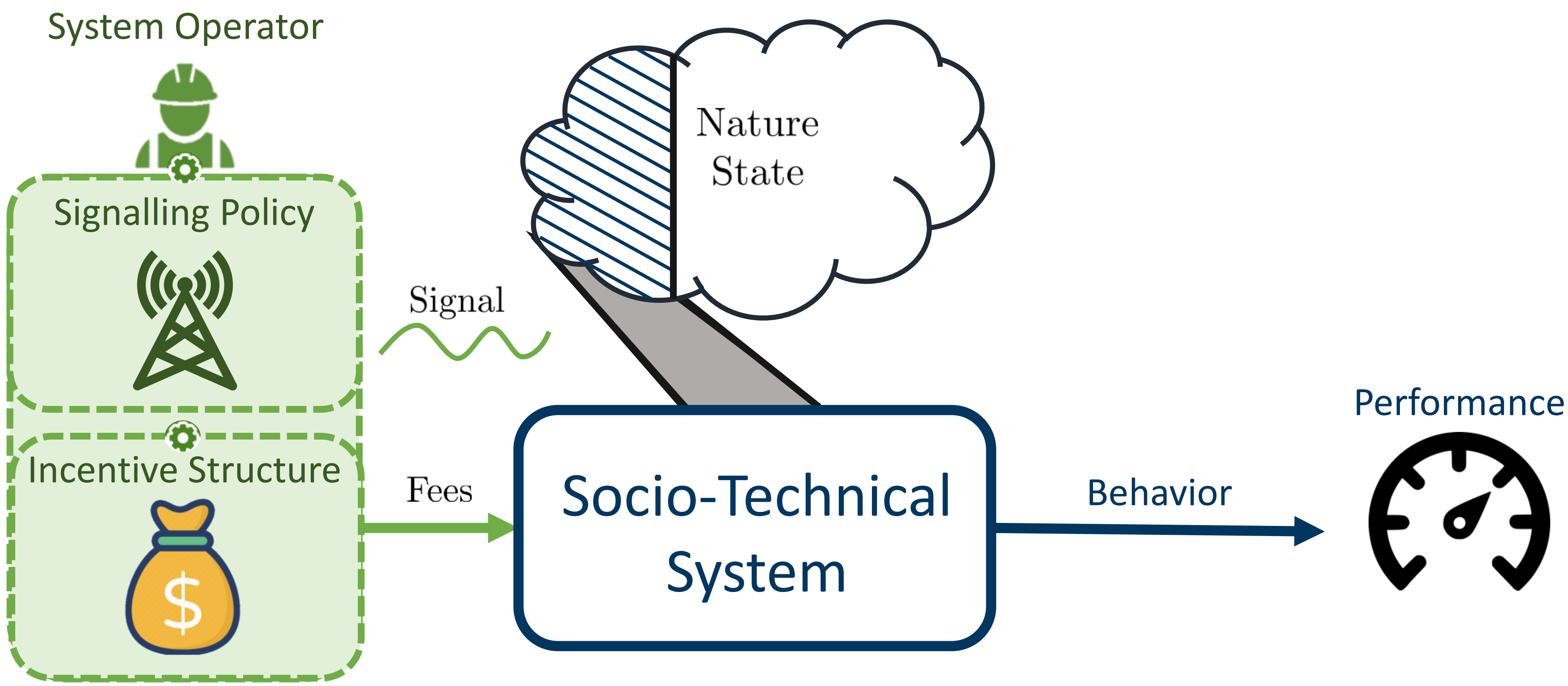
Signalling & Incentives

Co-design

$$\min_{\pi} \ \mathcal{L}(f;\pi)$$

$$\min_{\pi,T} \ \mathcal{L}(f;\pi,T)$$

$$\min_{T} \ \mathcal{L}(f;T)$$



Can ***co-designing*** mechanisms improve performance?

Signal-aware incentive mechanism

$$T(m) = \{\tau_e(m)\}_{e\in E}$$

Signal-Aware Incentive Design

Co-designed Mechanisms

Design together → Better performance than individual designs

Signal-aware incentive mechanism

$$T(m) = \{\tau_e(m)\}_{e \in E}$$

Proposition 3.1: For a signalling policy π, the optimal signal aware incentive $T^\star$ assigns incentives

$$\tau_e^\star(m) = \sum_{d \in \mathcal{D}} \mathbb{E}[\alpha_{e,d} | m] \cdot z_e \cdot \ell_d'(z_e)$$

where $z \in \arg\min_f \mathcal{L}(f; \mathbb{E}[\alpha|\pi_i])$.

Co-design → Single design

$$\min_{\pi, T} \ \mathcal{L}(f; \pi, T) \qquad \min_{\pi} \ \mathcal{L}(f; \pi, T^\star(\pi))$$

Signalling with Concurrent Incentives

Can signalling ***help***?

Can signalling ***hurt***?

Theorem 3.2: While using the signal-aware incentive policy $T^\star$, any signalling policy $\pi : A \to \Delta(M)$ has non-negative benefit to system cost, i.e.,

$$\mathbf{B}(\pi; \mu_0, T^\star) \geq 0 \quad \forall G, \mu_0, \pi.$$

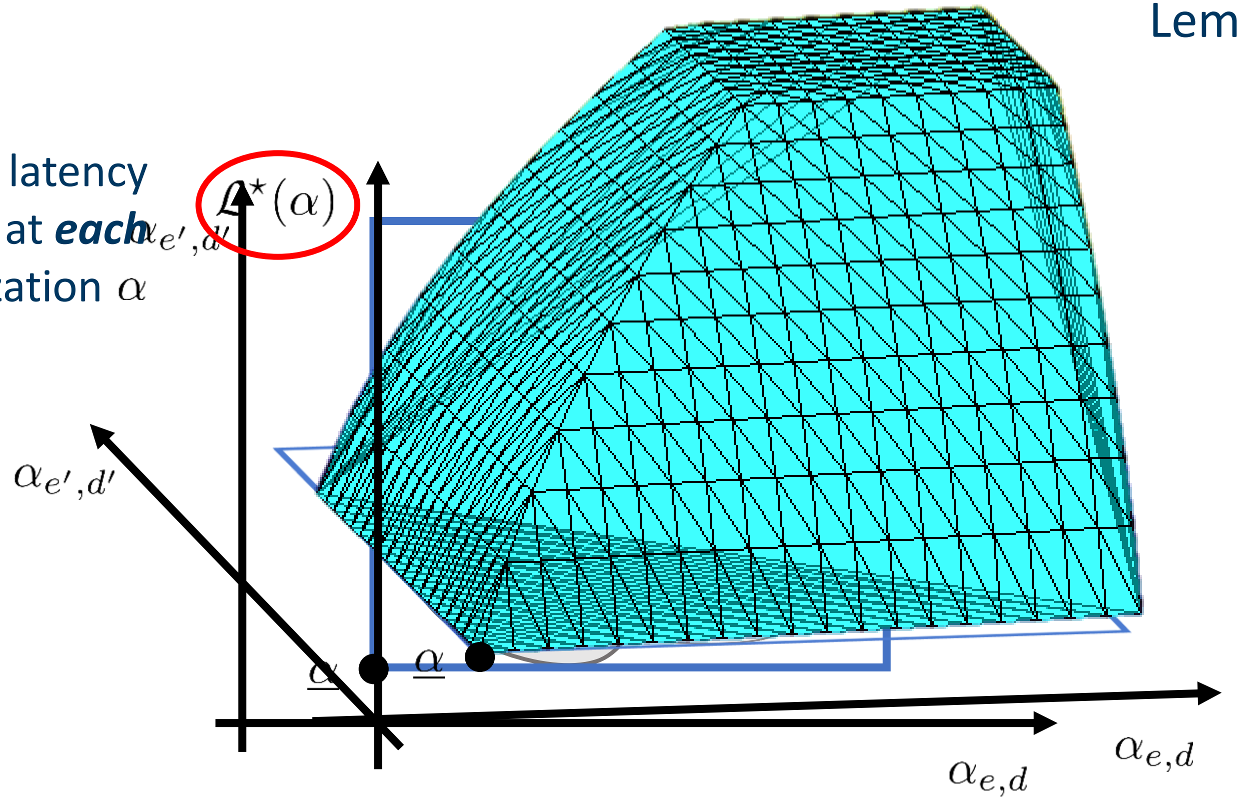
Recall:

$$\mathbf{B}(\pi; \mu_0) \geq 0 \quad \Rightarrow \quad \mathcal{L}^{\mathrm{BNf}}(\pi; \mu_0) \leq \mathcal{L}^{\mathrm{BNf}}(\emptyset; \mu_0)$$

Signalling can never be bad
when we use incentives

Proof Sketch

$\mathcal{L}^\star(\alpha) :=$ Total Lat. in Nash flow with $T^\star$



Recall

Lemma: $\mathcal{L}^{\mathrm{BNf}}(\pi;\mu_0,T^\star) = \sum_{m\in M} \psi(m)\mathcal{L}^\star(\mathbb{E}_{\alpha\sim\mu_m}[\alpha])$

$$\mathcal{L}^\star(\alpha) = \inf_{f\in\mathcal{F}(G)} \mathcal{L}(f,\alpha) = \sum_{e\in E}\sum_{d\in\mathcal{D}} \alpha_{e,d}\ell_d(f_e)$$

concave ← Affine in α

Convex hull of
the graph of $\mathcal{L}^\star(\alpha)$

No gap above → No negative benefit

$$\mathcal{L}^{\mathrm{BNf}}(\pi;\mu_0,T^\star) = \sum_{m\in M} \psi(m)\mathcal{L}^\star(\mathbb{E}_{\alpha\sim\mu_m}[\alpha])$$
$$\leq \mathcal{L}^\star\left(\sum_{m\in M} \psi(m)\mathbb{E}_{\alpha\sim\mu_m}[\alpha]\right)$$
$$= \mathcal{L}^\star(\mathbb{E}_{\mu_0}[\alpha])$$
$$= \mathcal{L}^{\mathrm{BNf}}(\emptyset;\mu_0,T^\star)$$

Benefit of Signalling with Incentives

How much can signalling ***help***?

Restrict to parallel networks and polynomial latency functions

i.e., $\ell_e(f_e) = \sum_{d\in\mathcal{D}} \alpha_{e,d}(f_e)^d \qquad \mathcal{D} = \{d_1, \dots, d_{|\mathcal{D}|}\},\ d_i \in \mathbb{N}$

Theorem 3.3: For any set of polynomial degrees $\mathcal{D}$, prior μ_0, and signalling policy π:

$$0 \le \mathbf{B}(\pi; \mu_0, T^\star) \le \sqrt{|\mathcal{D}|} \cdot \|\mathbb{E}[\alpha] - \underline{\alpha}\|_2,$$

where $\mathbb{E}[\alpha] = \int_{x\in A} x\cdot\mu_0(x)dx$, and $\underline{\alpha} \in \mathbb{R}_{\ge 0}^{|E|\cdot|\mathcal{D}|}$ such that $\underline{\alpha}_{e,d} = \inf\{\text{supp}(\alpha_{e,d})\}$ for each $e \in E$, $d \in \mathcal{D}$.

Observations:

1. With incentives, signalling *can only help*
2. Signalling still has the same capabilities to improve performance

Insights on Signalling with Incentives

- Incentives make signalling robust
  - No negative benefit

- Signalling maintains similar improvement capabilities

Q?: How do we ***design*** signals?

No signals	Full reveal/truthful	Optimal signals (w/ and w/o incentives)
$\pi = \emptyset$	$\pi(\alpha) = \alpha$	$\pi^\star$

Optimal Signals *without* Incentives

Parallel networks and polynomial latency

i.e., $\ell_e(f_e) = \sum_{d \in \mathcal{D}} \alpha_{e,d}(f_e)^d$

Finite support

$A = \{\alpha^1, \ldots, \alpha^{|A|}\}$

Decision variables: $\pi \in \mathbb{R}^{|A| \times |A|}$ where $\pi(m,k) = \mathbb{P}[m|\alpha^k]$

$\mathbf{f} \in \mathbb{R}^{|E| \times |A|}$ where $\mathbf{f}(e,m) =$ flow on edge e with signal m

Objective:

$$\min_{\mathbf{f} \in \mathbb{R}_{\geq 0}^{|E| \times |A|},\ \pi \in \mathbb{R}_{\geq 0}^{|A| \times |A|}} \quad \mathcal{L}(\mathbf{f}; \mu_0, \pi) = \sum_{k=1}^{|A|} \sum_{m=1}^{|A|} \sum_{e \in E} \sum_{d \in \mathcal{D}} \alpha_{e,d}^k (\mathbf{f}(e,m))^{d+1} \cdot \pi(m,k)\mu_0(k)$$

$$\text{s.t.} \quad \mathbf{f}(e,m) \cdot \sum_{k=1}^{|A|} \left(\ell_e^k(\mathbf{f}(e,m)) - \ell_{e'}^k(\mathbf{f}(e',m))\right)\pi(m,k)\mu_0(k) \leq 0 \quad \forall e, e' \in E,\ m \in M$$

$$\mathbb{1}_{|E|}^T \mathbf{f}(-,m) = r \cdot \mathbb{1}_{|A|}^T$$

$$\mathbb{1}_{|A|}^T \pi = \mathbb{1}_{|A|}^T$$

Eq. constraint

Polynomial objective/ constraints

Cast as GMP → Approx. sol. w/ SDP [Zhu, et. al.]

Optimal Signals *with* Incentives

Parallel networks and polynomial latency

i.e., $\ell_e(f_e) = \sum_{d \in \mathcal{D}} \alpha_{e,d} (f_e)^d$

Finite support

$A = \{\alpha^1, \ldots, \alpha^{|A|}\}$

Decision variables: $\pi \in \mathbb{R}^{|A| \times |A|}$ where $\pi(m,k) = \mathbb{P}[\alpha^k | m]$

$\mathbf{f} \in \mathbb{R}^{|E| \times |A|}$ where $\mathbf{f}(e,m) =$ flow on edge e with signal m

Flow optimal at each signal

$$\min_{\mathbf{f} \in \mathbb{R}^{|E| \times |A|}_{\geq 0},\ \pi \in \mathbb{R}^{|A| \times |A|}_{\geq 0}} \mathcal{L}(\mathbf{f}; \mu_0, \pi, T^\star) = \sum_{k=1}^{|A|} \sum_{m=1}^{|A|} \sum_{e \in E} \sum_{d \in \mathcal{D}} \alpha^k_{e,d} (\mathbf{f}(e,m))^{d+1} \cdot \pi(m,k) \mu_0(k)$$

s.t.

~~$$\mathbf{f}(e,m) \cdot \sum_{k=1}^{|A|} (\ell^k_e(\mathbf{f}(e,m)) - \ell^k_{e'}(\mathbf{f}(e',m))) \pi(m,k) \mu_0(k) \leq 0$$~~

$$\forall e, e' \in E,\ m \in M$$

Eq. constraint

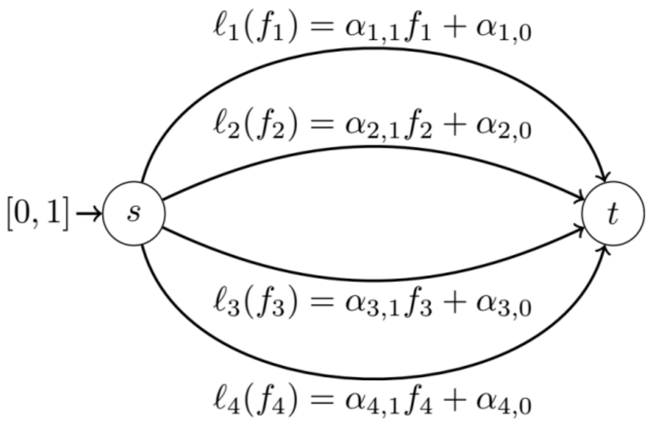
$$\mathbb{1}^T_{|E|} \mathbf{f}(-,m) = r \cdot \mathbb{1}^T_{|A|}$$

$$\mathbb{1}^T_{|A|} \pi = \mathbb{1}^T_{|A|}$$

Posynomial objective/ constraints

Cast as Geo. program → Solve as convex problem

Numerical Result



Latency Functions	
Uncongested	Congested
$\ell_1(f_1) = 25f_1 + 5$	$\ell_1(f_1) = 30f_1 + 25$
$\ell_2(f_2) = 17f_2 + 10$	$\ell_2(f_2) = 35f_2 + 13$
$\ell_3(f_3) = 13f_3 + 15$	$\ell_3(f_3) = 25f_3 + 20$
$\ell_4(f_4) = 10f_4 + 25$	$\ell_4(f_4) = 11f_4 + 35$

State Distribution	
State 1: w.p. 0.3	Uncongested: e_1, e_2, e_4 Congested: e_3
State 2: w.p. 0.4	Uncongested: e_3, e_4 Congested: e_1, e_2
State 3: w.p. 0.3	Uncongested: e_2 Congested: e_1, e_3, e_4

Info./Incentive Setting	System Cost
No signal / No tolls	23.99
True signal / No tolls	22.25
Opt. signal / No tolls	21.91*
No signal / w/ tolls	23.41
True signal / w/ tolls	21.30
Opt. signal / w/ tolls	21.29*

Insights:

- Optimal design helps
- Co-design gives best performance
- Revealing truth is good with incentives

Summarizing Remarks

Monetary Incentives

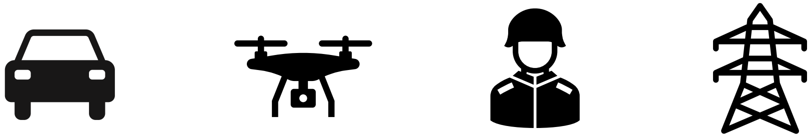
Information Signalling

Co-designed Mechanisms

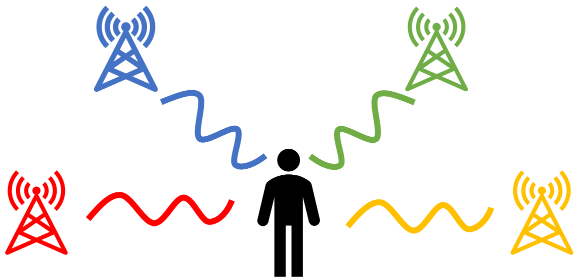
- ***Information*** is valuable in incentive design
  - Subsidies and tolls
- Signalling information can be ***helpful*** or ***hurtful***
- Signal/incentive co-design makes signalling ***robust***
  - and leads to best performance

Future Direction

- Signalling in Other Domains



- Multiple Senders



- Non-Bayesian Receivers

UC SANTA BARBARA

Information and Influence: Overcoming and Exploiting Uncertainty in Congestion Games

Bryce L. Ferguson

In collaboration with Jason R. Marden and Philip N. Brown

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